Chapter 2

Investigation of the Instability Ingredient in Mid-latitude Cyclones

Instability in winter season weather systems has recently been the subject of increased attention among research and operational communities (e. g., Wiesmueller and Zubrick, 1998; Holsten and Hendricks, 1997; Nicosia and Grumm, 1999; SS). However, precipitation events modulated by instability remain one of the most poorly forecasted wintertime events. This chapter investigates the nature of moist instability in mid-latitude cyclones and discusses appropriate techniques for forecasting the release of gravitational or symmetric instability in a winter precipitation event. An analysis of the mechanisms responsible for the evolution of instability throughout the life cycle of a mid-latitude cyclone is presented, including a diagnosis of geostrophic equivalent and saturated geostrophic equivalent potential vorticity, PV_{e_g} and PV_{es_g}, respectively.

The adiabatic and frictionless quasi-geostrophic ω equation (Bluestein, 1992) allows for the instantaneous diagnosis of vertical velocity (ω) based on geopotential (ϕ) and temperature (T). This equation can be written:

$$\left(\sigma\nabla_p^2 + f^2\frac{\partial^2}{\partial p^2}\right)\omega \quad = \quad -\frac{R}{P}\nabla_p^2\left(-\vec{V_g}\cdot\nabla_p T\right) \quad + \quad f\frac{\partial}{\partial P}\left[\vec{V_g}\cdot\nabla_p\left(\xi_g + f\right)\right] \tag{2.1}$$

term 1 term 2 term 3

where $\omega = \frac{\partial p}{\partial t}$, $\sigma = -\frac{RT}{P\theta}\frac{\partial\theta}{\partial P}$, $\nabla_p = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$ on a pressure surface, f is the Coriolis parameter, p is pressure, R is the gas constant for dry air, $\vec{V_g}$ is the geostrophic wind ($\vec{V_g} = \frac{1}{f}\,\hat{k}\times\nabla_p\,\phi$), and ξ_g is the vertical component of the geostrophic relative vorticity vector, $\xi_g = \hat{k}\cdot\left(\nabla\times\vec{V_g}\right)$. Terms 2 and 3 represent the quasi-geostrophic forcing for vertical motion, and have traditionally been considered as the separate physical processes of the Laplacian of horizontal temperature advection and the rate of change of vorticity advection with height. Term 1 is similar to a three-dimensional Laplacian of ω, $\nabla^2\omega = \left(\frac{\partial^2\omega}{\partial x^2}, \frac{\partial^2\omega}{\partial y^2}, \frac{\partial^2\omega}{\partial p^2}\right)$, except that this pseudo-Laplacian operator is modulated by a stability parameter σ and the Coriolis parameter f.

Equation 2.1 illustrates that the response (ω) to a given forcing is inextricably tied to atmospheric stability (σ). For a given forcing (terms 2 and 3) and a small value of σ, $\nabla^2\omega$ must be large (provided ω has an x or y dependence). A large $\nabla^2\omega$ implies enhanced differences between the maxima and minima in the ω field and either increased vertical motions or a decreased horizontal scale of the vertical motion field (Bluestein, 1992). Thus, the stability of the atmosphere in an area of forcing for vertical motion affects both the location and the intensity of the subsequent vertical motion.

2.1 Types of Moist Instability

An important distinction in evaluating moist instability is whether it is conditional or potential (convective) instability. Rogers and Yau (1989) note, “Convective [potential] instability has to do with the lifting of layers and should not be confused with conditional instability which applies to an undisplaced layer.” An air column that is conditionally unstable requires only an infinitesimal displacement to realize the instability, provided that the air is saturated locally. An air column that is potentially (convectively) unstable requires a finite vertical displacement to reach saturation and realize the instability (Schultz and Schumacher, 1999; hereafter SS).

Instability can be further distinguished as gravitational, inertial, and symmetrical based on the direction in which a displaced air parcel is compelled to move when the instability is realized. With gravitational instability, an air parcel accelerates vertically once displaced up or down. With inertial instability, an atmosphere is similarly unstable to horizontal displacements. Inertial instability occurs when the absolute vorticity is negative, a condition that is only occasionally observed in mid-latitudes and thus will not be considered in this thesis. Symmetrical instability exists in a two-dimensional atmosphere when it is stable to both vertical and horizontal displacements is unstable to displacements along slanted paths. These classifications of instability can be combined to distinguish four types of moist instability: conditional gravitational instability (CI), conditional symmetric instability (CSI), potential gravitational instability (PI), and potential symmetric instability (PSI). These types are summarized in Table 2.1.

	Conditional (θ_{es})	Potential (θ_e)
Gravitational	CI	PI
Symmetric	CSI	PSI

Table 2.1: Summary of four types of moist instability: conditional gravitational (CI), conditional symmetric (CSI), potential gravitational (PI), and potential symmetric (PSI) instabilities.

An atmosphere is characterized by CI if the environmental lapse rate (γ) is greater than the pseudoadiabatic (or saturated adiabatic) lapse rate (Γ_m) and less than the dry adiabatic lapse rate (Γ_d): $\Gamma_m < \gamma < \Gamma_d$ (Rogers and Yau, 1989). This requirement is equivalent to the condition that the saturated equivalent potential temperature (θ_{es}) decreases with height, $-\frac{\partial \theta_{es}}{\partial p} < 0$. CSI exists when the environmental lapse rate along a surface of constant absolute geostrophic momentum ($M_g = V_g + fx$, where the y-direction is taken to be in the direction of the geostrophic vertical shear) lies between the moist and dry adiabatic lapse rates (i. e., the atmosphere is conditionally unstable along an M_g surface). For a two dimensional atmosphere, CSI can be diagnosed where contours of θ_{es} are more steeply sloped than contours of M_g in a cross-section drawn perpendicular to the thermal wind (i. e., the $M_g - \theta_{es}$ relationship, e. g., Snook, 1992).

PI and PSI can be diagnosed similarly to CI and CSI, except that equivalent potential temperature (θ_e) is substituted for θ_{es}. Thus, an atmospheric layer is potentially unstable where $-\frac{\partial \theta_e}{\partial p} < 0$, and PSI is diagnosed by the $M_g - \theta_e$ relationship. Note that if the atmosphere is locally saturated, $\theta_e = \theta_{es}$ and potential and conditional instabilities are equivalent (PI $\equiv$ CI and PSI $\equiv$ CSI).

Because an atmospheric layer in an unsaturated potentially unstable atmosphere (PI or PSI) must undergo a finite vertical displacement to reach saturation and release the instability, potential instability is not a sufficient condition for moist convection. The magnitude of the required displacement will vary considerably under different thermodynamic conditions, and many regions of potential instability are never realized. In contrast, conditional instabilities (CI or CSI) only require an infinitesimal displacement and local saturation to be realized. Therefore, conditional instabilities are a more "appropriate measure of the susceptibility of the atmosphere to [gravitational and symmetric] convection" (SS).

One drawback to evaluating instability using the $M_g - \theta_e$ and $M_g - \theta_{es}$ relationships is that developing cross-sections can be cumbersome and time-consuming. In addition, this approach requires an atmosphere which can be accurately described in two dimensions. A more convenient and also more flexible technique for diagnosing moist instability on isobaric surfaces involves the use of geostrophic equivalent potential vorticity (PV_{e_g}) and saturated geostrophic equivalent potential vorticity (PV_{es_g}). The application of PV_e and PV_{es} to diagnose moist instabilities in cyclones is discussed in the next section.

2.2 Diagnosing Moist Instability

Geostrophic equivalent potential vorticity (PV_{e_g}) is defined as

$$PV_{e_g} = -g\, \vec{\zeta_g} \cdot \nabla \theta_e \tag{2.2}$$

where g is the gravitational constant, $\vec{\zeta_g}$ is the three-dimensional absolute geostrophic vorticity vector, ∇ is the gradient operator in x, y, and p coordinates, and θ_e is the equivalent potential temperature. PV_{e_g} has traditional potential vorticity units, PVU, where 1 PVU $= 1 \times 10^{-6} m^2 K s^{-1} kg^{-1}$. For a zonal flow, Martin et al. (1992) showed that the equivalent potential vorticity is

$$PV_{e_g} = \frac{\partial M_g}{\partial p}\frac{\partial \theta_e}{\partial x} - \frac{\partial M_g}{\partial x}\frac{\partial \theta_e}{\partial p} \qquad (2.3)$$

Regions of symmetric or gravitational instability can be identified where the quantity PV_{e_g} is negative. Use of the two-dimensional PV_{e_g} (equation 2.3) for this diagnosis, however, is limited to vertical cross-sections that meet the following conditions (SS): (1) the geostrophic wind direction doesn't vary with height (i. e., it is two-dimensional); (2) the cross-section is perpendicular to the thermal wind (the shear of the geostrophic wind), and (3) the along-flow ageostrophic wind is small so that the assumption of a geostrophic wind is reasonable. This excludes, for example, regions of curvature (violation of the 2-D assumption) or wind acceleration (violation of the assumption of small along-front ageostrophy). In cyclones, flow is rarely constrained to two dimensions, particularly in the occluded quadrant (the area northwest of the sea level pressure minimum where many traditional forecasting methods suggest heavy snow often falls) because the flow is highly curved. In areas such as fronts where the along-front variations are negligible and the flow is nearly two-dimensional, the three conditions listed above may occasionally be met. However, even in cases characterized by two-dimensional dynamics, the use of two-dimensional PV_{e_g} to diagnose instability still requires the tedious inspection of multiple cross-sections to assess the horizontal extent of instability, and the results are dependent on the orientation of the cross-section.

McCann (1995) observed that the complete three-dimensional PV_{e_g} (equation 2.2) can be used to identify regions of general instability without assuming a two-dimensional flow. By recognizing that the geostrophic wind shear $-\frac{\partial V_g}{\partial p}$ is related to the temperature gradient through the thermal wind balance, he showed that

$$PV_{e_g} = g\left(-\frac{1}{f\rho\theta_e}\frac{\Gamma_m}{\Gamma_d}|\nabla_p\theta_e|^2 - \hat{k}\cdot\vec{\zeta}_g\frac{\partial\theta_e}{\partial p}\right) \tag{2.4}$$

where Γ_d and Γ_m are the dry and adiabatic lapse rates, respectively. The first term on the right hand side is always negative, and its magnitude is determined by the horizontal temperature gradient. The second term is a measure of gravitational instability. McCann considered three possible scenarios. If the second term is negative (unstable to moist convection, $-\frac{\partial\theta_e}{\partial p} < 0$), PV_{e_g} is negative and vertical (gravitational) instability dominates regardless of the magnitude of the first term. If the second term is positive (stable to moist convection, $-\frac{\partial\theta_e}{\partial p} > 0$) and the temperature gradient is weak enough that the first term is smaller than the second, then PV_{e_g} is positive and the environment is stable. However, if the lapse rate is slightly stable ($-\frac{\partial\theta_e}{\partial p}$ only slightly greater than zero) and the horizontal temperature gradient is strong, the second term is positive but of lesser magnitude than the first term so that PV_{e_g} is negative. Under these conditions the result is slantwise (symmetric) instability. Based on these observations, McCann concluded that the three-dimensional PV_{e_g} combines gravitational and symmetric instabilities and becomes an all-purpose convection potential tool.

Use of a three-dimensional PV_{e_g} permits a quasi-horizontal analysis of instability on pressure surfaces and a cross-sectional analysis independent of the orientation of the cross-section. However, since symmetric instabilities remain strictly two-dimensional phenomena, the three-dimensional PV_{e_g} does not offer any information about symmetric instabilities *unless* the flow is two-dimensional. In contrast, gravitational instability can be diagnosed by negative PV_{e_g} (provided $-\frac{\partial\theta_e}{\partial p} < 0$) regardless of the curvature of the flow.

With virtually universal access to gridded data analysis programs (e. g., GEMPAK, AWIPS, Vis5D, GARP, NTRANS), the $M_g-\theta_e$ relationships and PV_{e_g} have increasingly been employed in operational forecasting and research for the diagnosis and evaluation of CSI (e. g., Snook, 1992; Moore and Lambert, 1993; Wiesmueller and Zubrick, 1998). However, SS identified a misconception behind the conventional application of PV_{e_g} to diagnose CSI. The authors noted that unless the air parcel is saturated, PV_{e_g} is strictly an indicator of potential instabilities (PI or PSI), and not conditional instabilities such as CSI. Instead, saturated equivalent geostrophic potential vorticity (PV_{es_g}) should be used to diagnose CI and CSI. PV_{es_g} is defined in the same manner as PV_{e_g} but with saturated equivalent potential temperature θ_{es} substituted for θ_e:

$$PV_{es_g} = -g\ \vec{\zeta_g} \cdot \nabla\theta_{es} \tag{2.5}$$

Saturated equivalent potential temperature is simply the equivalent potential temperature that an air parcel (at a given air temperature T and pressure P) would have if it were saturated at that same T and P. Mathematically,

$$\theta_{es} = \theta e^{\frac{L}{C_p}\frac{l_s}{T}} \tag{2.6}$$

where L is the latent heat of vaporization, l_s is the saturation mixing ratio at T and P, and C_p is the specific heat capacity of dry air at constant pressure.

Bennetts and Hoskins (1979) noted when they introduced their theory applying wet-bulb potential vorticity (analogous to PV_{e_g} in this context) to the diagnosis of CSI that such an approach is only valid in the presence of saturation. This assumption has often been neglected in the subsequent use of PV_{e_g} to diagnose CSI. At saturation, $\theta_e = \theta_{es}$ and $PV_{e_g} = PV_{es_g}$, and either quantity can be used to diagnose CSI. However, when conditions are unsaturated, the quantities are not equal and cannot be used interchangeably. SS pointed out the misuse of PV_{e_g} resulting from inappropriate applications of the original theory by Bennetts and Hoskins (1979) and suggested that "in the future, we, as a meteorological community, use the term CSI only when employing θ_e^* [their notation for θ_{es}] and use the term PSI only when employing θ_e." The effort of SS to reach the operational and academic communities is reflected in the increasing use of PV_{es_g} instead of PV_{e_g} (or $M_g - \theta_{es}$ instead of $M_g - \theta_e$) in the literature and among operational forecasters to diagnose CI and CSI (e. g., Nicosia and Grumm, 1999; Clark and Nicosia, 1999; Banitt, 1999; McCann 1999).

Hereafter, the moist potential vorticity quantities will be abbreviated as PV_e and PV_{es} for geostrophic equivalent potential vorticity and geostrophic saturated equivalent potential vorticity, respectively. Notice that although the "g" has been omitted from the abbreviation, geostrophic absolute vorticity is used in the calculation of these quantities.